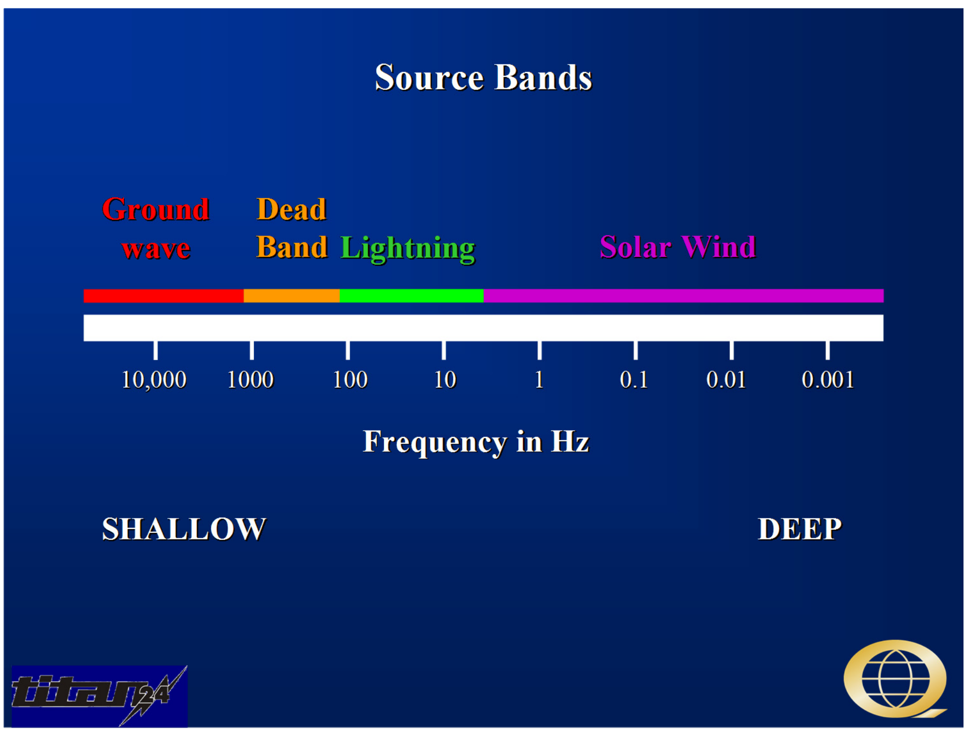
APPENDIX C - MAGNETOTELLURIC (MT) THEORY

The magnetotelluric (MT) method measures time-variations in the Earth's natural electric (E) and magnetic (H) fields to image the subsurface resistivity structure. No source or transmitter is used. These natural fields penetrate much deeper than is practical with a transmitter. At the same time the natural signals are a plane-wave source. The plane-wave source is much simpler to model than complex transmitter geometries and signals.

The E and H fields are measured over a broad range of frequencies. Typically, the frequencies can range from above 10 kHz to below 0.001Hz. High frequency signals are attenuated more rapidly in the subsurface. High frequency data are indicative of shallow resistivity structure while low frequency data are indicative of deep resistivity structure.

At frequencies below 1Hz the signal source is due to oscillations of the Earth's ionosphere as it interacts with the solar wind. At frequencies above 1Hz the signal source is due to worldwide lightning activity. There is a lack of signal around 1Hz, often referred to as the "hole". Modern 24-bit recording hardware and signal processing techniques have largely eliminated the data quality problems that have been traditionally seen around the 1Hz signal hole.

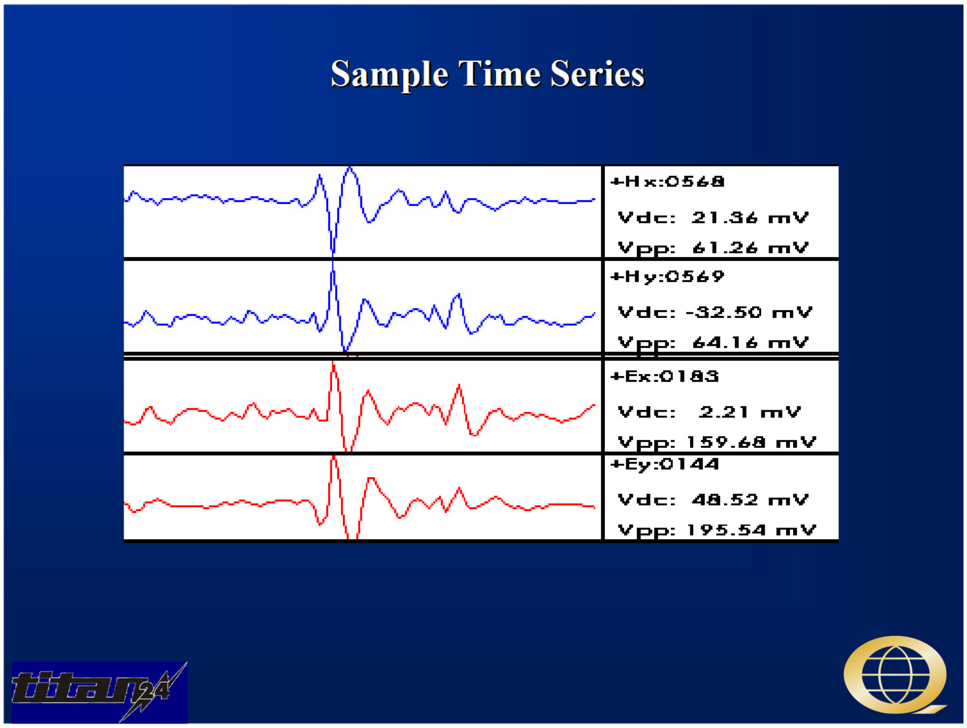
Between about 8Hz and 300Hz the signal from worldwide lightning activity propagates in a "resonant" cavity (the resistive atmosphere) between the conductive ionosphere and the Earth's surface. Above 3 kHz the signal propagates as a ground wave. Between 300Hz and 3 kHz there is a "dead-band" where the signal does not propagate well. Despite hardware and signal processing improvements this dead-band remains problematic. When signal (atmospheric activity) is present within several hundreds of miles of the survey area the data is quite good. When no signal is being generated in the vicinity data quality is poor.



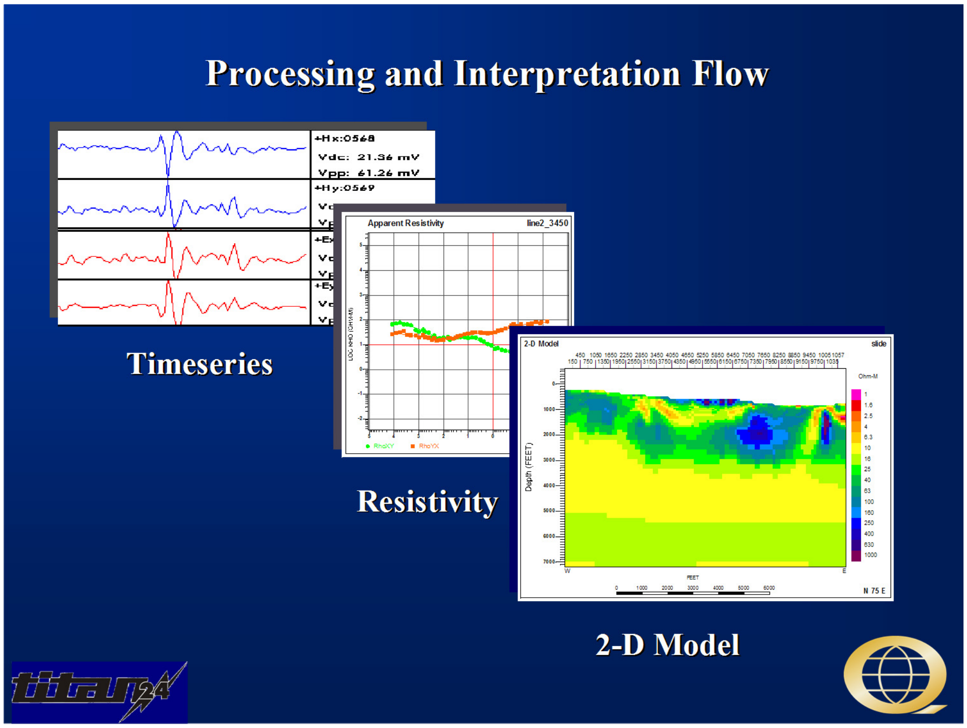
Both the electric and magnetic fields are measured. The measured fields depend on the ionosphere and lightning, and are essentially random. While the E and H fields are random the ratio of the fields depends on the subsurface resistivity structure. Note that it is primarily the orthogonal E and H fields that are related. The magnetic field must be measured perpendicular to the electric field. It is possible for complex subsurface resistivity structure to rotate the fields, and full tensor data are usually measured.

It is often useful to think of the magnetic field as the source signal and the electric field as the response. Time variations in the magnetic field induce currents to flow in the ground.

In the field the electric and magnetic fields are measured as a function of time. The electric field is measured using two orthogonal dipoles consisting of a wire connecting two grounded electrodes. In essence, the recording system consists of a voltmeter between the electrodes. The voltage measured depends on the electric field strength and the length of the dipole. The magnetic field is measured using an induction coil.



While the actual fields that are measured vary randomly (with solar and lightning activity), the relationship between the measured magnetic and electric fields is constant and depends on the subsurface resistivity structure. Extracting the subsurface resistivity structure from the measured magnetic and electric fields is a multi-step process. First time series processing techniques are used to derive geophysical parameters from the electric and magnetic fields. Then geophysical processing and inversion techniques are used to convert the geophysical parameters to a subsurface resistivity image. Finally, the resistivity image must be interpreted in terms of geologic units.



The measured magnetic and electric fields are Fourier transformed into the frequency domain. The system response is removed from the data (making the measurement independent of the hardware system). The Fourier coefficients represent the amplitude and phase of the electric and magnetic fields as a function of frequency.

A variety of signal processing techniques are used to minimize noise and bias in the estimation of geophysical parameters from the measured fields. The details are complex, but the approach is easily understood. Philosophically, the idea is to use multiple approaches to noise and bias reduction, not letting any one statistical approach have too much impact on the data, but relying on the combination of approaches to produce good estimates. The approaches include:

1. Spatial isolation of noise. A remote reference magnetic station is used to separate widely distributed signal from local noise.
2. Coherency sieves to find coherent signal. First the local and remote magnetic field measurements are compared and coherent signal kept. Then the local magnetic and electric fields are compared for coherency.
3. Frequency isolation of noise. Long Fourier transforms are used to provide extremely sharp isolation of noise in frequency.
4. Time isolation of noise. Short Fourier transforms are used to remove noise that is isolated in time (noise spikes, or noise that is randomly turning off and on).
5. Robust statistics that minimize biasing effects of a few isolated measurements.

Once the time series processing is complete geophysical parameters can be estimated. The primary geophysical parameters for MT are typically the apparent resistivity versus frequency and phase versus frequency.

The depth of penetration of the signal depends on its frequency and the resistivity of the rocks. The depth at which the signal amplitude has been attenuated to 37% (1/e) is called the skin depth and is defined:

$$\delta = \sqrt{\frac{2}{\mu\omega\sigma}} = 503\left(\sqrt{\frac{\rho}{f}}\right)(m)$$

where

δ = skin depth

μ = magnetic permeability

σ = conductivity=1/resistivity

ω = angular frequency=$2\pi f$

ρ=resistivity=1/conductivity

The ratio between the two measured components (E and H) is the electrical impedance. The impedance (denoted Z) is defined as $|Z|=|E/H|$. The impedance is a complex number because the E and H fields are out of phase. Note that Z, E, and H are all functions of frequency.

The complex impedance is used to calculate an apparent resistivity as follows:

$$\rho_a = \frac{1}{\mu\omega}|Z|^2\,(ohm.m)$$

The apparent resistivity is also a function of frequency. At any frequency the fields must travel through the overlying geology. The apparent resistivity depends on the integrated (weighted) conductance of the rocks being sampled. It is a smoothly varying function of frequency because it represents the average resistivity of a progressively larger volume of the subsurface. On a log resistivity-log frequency plot the apparent resistivity generally can not exceed a slope of +/- 45 degrees.

The phrase “apparent resistivity” arises from the volume averaging. At a single frequency the electric and magnetic fields measurements can be used to calculate an impedance. This impedance depends on the resistivity of a large volume of the subsurface. The impedance can be thought of as the impedance of a half-space that would provide identical measurements to the actual subsurface.

The calculated phase or apparent phase is the difference between the measured E field phase and the measured H field phase. If the subsurface is one-dimensional (1D) or two-dimensional (2D) the phase is related to the resistivity. The Hilbert formula (minimum phase wavelet) relates the phase to the slope of the apparent resistivity curve. If the slope of the resistivity curve (on a log-log plot) is 0 the phase is 45 degrees. If the resistivity is increasing with decreasing frequency the phase is less than 45 degrees. If the resistivity is decreasing with decreasing frequency the phase is more than 45 degrees. As the apparent resistivities are constrained to a slope of no more than 45 degrees on a log-log plot, the phases are constrained to remain in a quadrant, between 0 and 90 degrees.

The phase measurement is largely independent of the apparent resistivity measurement. The Hilbert relationship provides an independent way to calculate the apparent resistivity curve from the phase data. There are effectively two independent measurements of the resistivity curve, providing a powerful check on data quality.

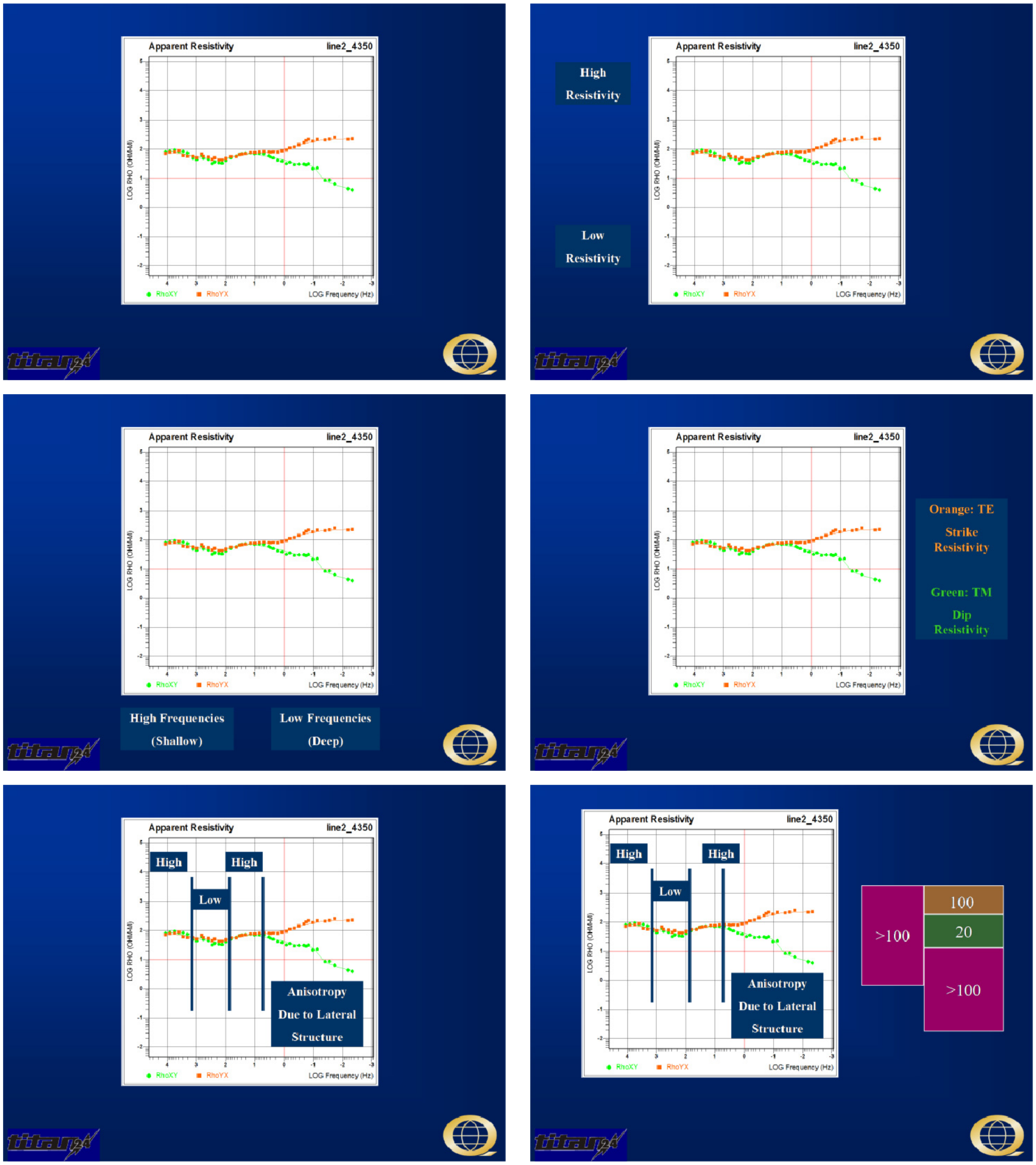
The apparent resistivity and phase curves are the primary parameters used in the interpretation of MT data. For a layered (1D) earth the apparent resistivity and phase data can be converted into intrinsic resistivity versus depth simply by accounting for the volume averaging nature of the method. There are a variety of algorithms for doing the conversion. The conversion is not unique. Some algorithms provide smoothly varying intrinsic resistivity versus depth functions (Occam inversion, Bostick transform). Others provide distinct layered solutions (Marquardt inversion).

1D modeling and inversion raises the following points:

- A single MT site provides information about resistivity versus depth. This is a major distinction from potential fields techniques that only provide information about relative variations along a profile.
- The conversion from apparent resistivity versus frequency to intrinsic resistivity versus depth is not unique. It is susceptible to equivalence. In particular any sharp resistivity contrast can be replaced by an equivalent transition zone.
- In a layered model the thickness of a resistive layer is well resolved. The resistivity of a resistive layer is poorly resolved.
- In a layered model the conductance (conductivity*thickness) of a layer is resolved. Neither the thickness nor the conductivity is uniquely resolved.
- Once the constraint that the subsurface is composed of distinct, resolvable, units is imposed the 1D inversion of MT data is essentially unique. Resolution is excellent (better than 5% of depth).

Apparent resistivity versus frequency is the most fundamental way of looking at the data in the interpretation phase. While the overall process is complex, with advanced processing techniques and inversions, it is important to keep in mind that the subsurface structures are apparent in the raw data – the apparent resistivity plots.

The following sequence of illustrations is intended to introduce the apparent resistivity versus frequency sounding curves. But it is also intended to highlight the relatively complex, but understandable, relationships between the observed data and subsurface structure.



A simple layered subsurface structure is not generally the problem of immediate interest in exploration. In the case of more complex two-dimensional (2D) or three-dimensional (3D) structure the MT response will be affected by lateral resistivity variations.

The MT measurement relies on natural, plane-wave, source signals. The measured response depends on lateral resistivity variations as much as (or more than) resistivity variations below the immediate sounding site.

Full tensor measurements of the E and H fields are made at every site. For each site there are two apparent resistivity sounding curves (or modes) and phase curves. These two modes are arbitrarily labeled Rho-XY and Rho-YX. The first, Rho-XY, refers to the apparent resistivity (Rho) calculated from Ex and Hy.

Once full tensor measurements are made in the field it is possible to mathematically rotate the fields to any arbitrary coordinate system. Traditionally, the data are rotated independently at each frequency to maximize the difference between the two apparent resistivity sounding curves. This puts the data into "geologic" or "principal" coordinates.

One sounding curve will have the electric field in the geologic strike direction and is referred to as "Transverse Electric" or TE. The other mode will have the electric field in the geologic dip direction and is referred to as "Transverse Magnetic" or TM. Note that TE and TM are interpretive designations, and refer to geologic strike. XY and YX were simply geometric designation.

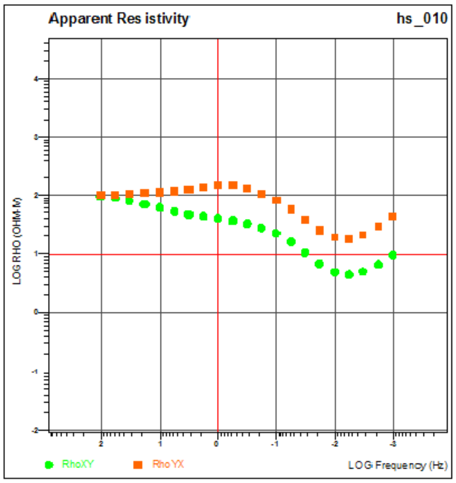
For a layered (1D) earth the two measurements are identical. When the structure is 2D or 3D the lateral resistivity variations will distort (often severely) the simple 1D response. The distortion of the fields by complex structure is realized in the apparent resistivity data as "anisotropy". This is a divergence between the two apparent resistivity sounding curves.

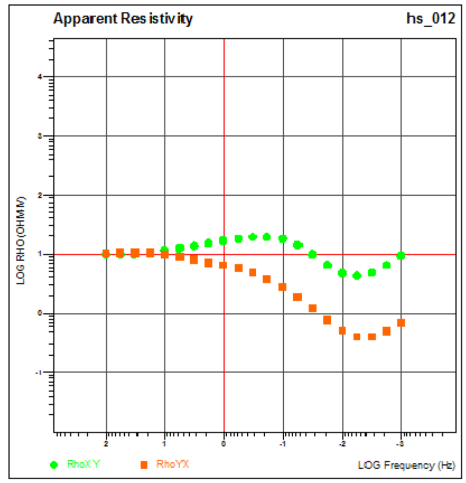
The measurement of two orthogonal apparent resistivity sounding curves provides valuable information. Both curves reflect the resistivity structure underlying the site. Both curves will show increasing or decreasing resistivity at a frequency in response to resistivity structure under a site. The two apparent resistivity curves will diverge in response to lateral resistivity variations.

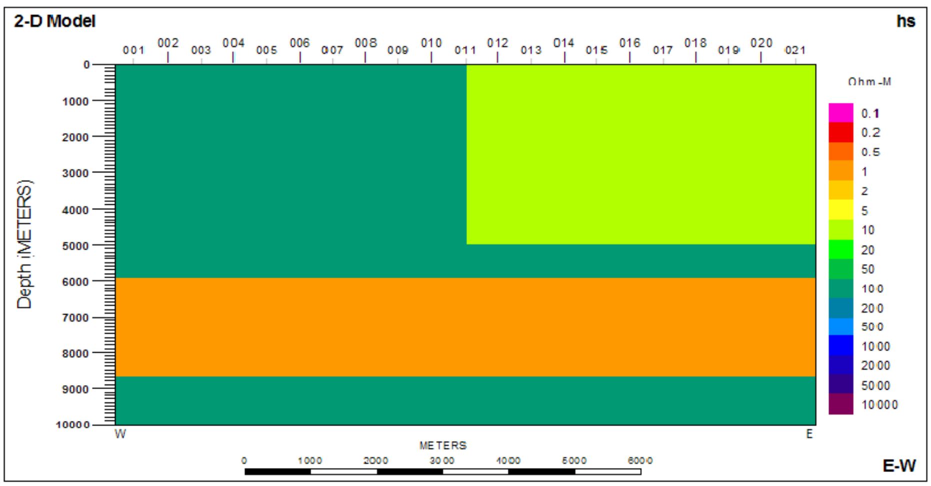
If the site is located on the resistive side of a lateral resistivity contrast the TE mode will be slightly suppressed due to the contact and the TM mode will be significantly biased up by the contact. If the site is located on the conductive side of a lateral resistivity contrast the TE mode will be slightly biased up while the TM mode will be significantly biased down by the contact.

For a 2D resistivity structure the TE mode is always providing an indication of the integrated conductance of the volume being sampled. It will always be a slowly varying function of position. The TM mode is responding dramatically to the presence of changes on the lateral resistivity boundaries, and will dramatically overshoot on the resistive side of a contact and undershoot on the conductive side. The anisotropy (divergence of the two sounding curves) is diagnostic of a lateral resistivity contrast.

The following simple model demonstrates most of the critical 2D behaviours. The model consists of a 100 Ohm-m host with a 10 Ohm-m basin on the right. There is a 1 Ohm-m layer buried within the host and below the basin. The response is shown at two sites, one immediately on the resistive side of the basin contact and the other immediately on the conductive side of the contact.



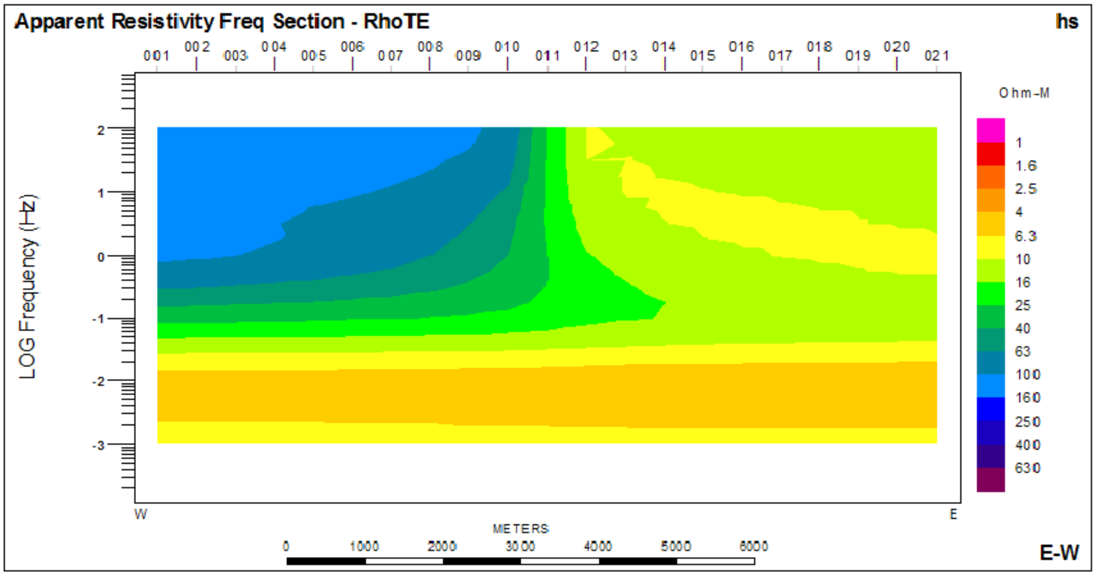


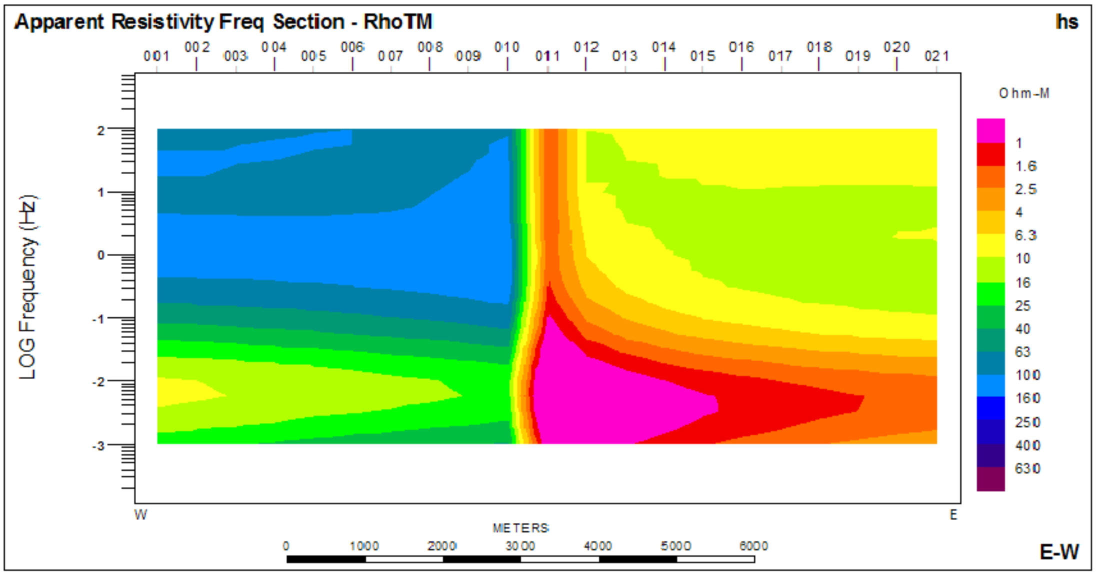


The following observations summarize the behaviour of 2D MT responses:

- The apparent resistivity at high frequencies reflects the true shallow resistivity.
- The apparent resistivities converge at high frequencies to the true shallow resistivity.
- The divergence in apparent resistivities occurs at a higher frequency for the site on the resistive side of the contact. Because the skin depth is larger in the resistive media the site on the resistive side of the contact is effectively “closer” to the contact than the site on the conductive side of the contact. The TE mode is constrained to the range of physical resistivities actually present in the model
- The TE mode “volume averages” the intrinsic resistivity. The TM mode exhibits apparent resistivities outside the range of physical resistivities in the model. Note that for the site on the right the TM mode indicates resistivities below 1 Ohm-m.
- Both the TE and TM modes respond in tandem, at the same frequency, to resistivity structure under the site. At both sites both the TE and TM modes indicate the top and bottom of the 1 Ohm-m layer. While 1D inversion of the TE and TM modes would place different apparent depths to the 1 Ohm-m layer the response is at the same frequency in both modes indicating it is the response of one layer.
- The intrinsic resistivity of the 1 Ohm-m layer is difficult or impossible to discern. Without physical property data only the conductance of the layer can be resolved.

These effects can also be clearly seen in pseudo-sections of the TE and TM apparent resistivity response of the model:





The apparent resistivity data at each site have been contoured, as a function of frequency. The inherent smoothness of the TE section can be clearly discerned. The distinctive "undershoot" of the TM response on the conductive side of the contact can be clearly seen.

One of the key factors in multidimensional MT data is "static shifts". The apparent resistivity sounding curves can be biased, up or down, by lateral resistivity contrasts too small to be resolved by the MT data. The curve is essentially DC shifted on the log-log apparent resistivity plot. This can be seen by examining the sounding curves from the previous 2D model. Assuming data had not been acquired above 1Hz the two sounding modes would be seen to be separated in the highest frequency data. Note that there are no static shift effects in the phase data.

Inversions and forward modeling are used to derive the subsurface resistivity structure from the data. The primary interpretation tools are 2D inversions. Problems emerge when the real world, complex, data are not consistent with the simplistic 2D assumptions. In a perfect world we would use modeling and inversion programs capable of reflecting the full complexity of the subsurface. However, in practice incorporating too much complexity in the modeling and inversion programs results in very coarse models which are incapable of resolving exploration targets. Instead, we must find ways to remove some of the complexity from the actual data. To this end, we have developed the Titan "EVA" data processing stream:

- Rotation to principal coordinates. The inversion algorithms presume that we have acquired a true geologic dip profile. In reality, geologic dip is often difficult to define, and seldom known prior to acquisition. However, because we have acquired full tensor data we can rotate our data to the geologic dip direction after acquisition.
- Eigenvector processing. 3D structures can introduce complex "rotations" of the electrical currents. These rotations produce effects, such as excessively steep resistivity curves and out-of-range phases, which would be impossible to fit with 2D modeling programs. By relaxing the assumption that the electric and magnetic fields are orthogonal, eigenvector analysis provides a unique and trivial methodology for simplifying complex 3D data.
- 1D inversion for curve fitting. Real data are often noisy, and inconsistent. Out-of-range phases are a typical example of features seen in real data that can not be fit using 2D inversion. It is often best to make use 1D inversion to make interpretative decisions about how to "best" fit the data, rather than letting the 2D inversion thrash trying to fit inconsistent data.

Once these data processing techniques have been completed the data are inverted. Generally, two inversions of the MT data are done. The first inversion uses an approach (a model norm) that explicitly looks for the "smoothest" model consistent with the data. This approach essentially finds the minimal subsurface structure consistent with the data. The second inversion uses an approach (a model norm) that looks for a model most consistent with the known geology.

For the geologically constrained inversion we use a proprietary approach developed by Dr. Phil Wannamaker. This approach uses the geologic constraints as a target, while not imposing any intrinsic smoothing on the inversion. The approach finds the maximum structural information, at the risk of sometimes including structure not required by the data. It represents an effort to extract the maximum exploration information from the data.

Both approaches are valid, and important. A smooth model approach to inversion can be viewed as finding the least possible useful exploration information. However, it does provide an independent assessment of what the data actually require. The geologically constrained inversion will provide a much sharper subsurface image. But it will also reproduce the known geology where the data does not require a change to the model. Without an independent smooth model inversion it can be hard to determine whether a geologically constrained inversion has confirmed the geologic interpretation, or simply doesn't have any information either way.